On the stability of Quantum Hadro-Dynamics

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Abstract

We explore the possible occurrence of σ-ω condensation in the Quantum Hadro-Dynamics (QHD), namely the Serot and Walecka model, finding that at the mean field level it corresponds to a critical value of the coupling constant $g_\sigma = 8.828$ and density $k_F = 207.2$ MeV/c, significantly below the standard value of QHD.

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1 Introduction

The Quantum Hadro-Dynamics (QHD) is a largely used model to describe nuclear systems within a coherent and covariant frame. Its parameters (σ and ω masses and coupling constants) are tuned to reproduce the energy and density of the nuclear matter at the mean field level, where exotic or non-trivial phenomena are intrinsically forbidden.

In going beyond the mean field, however, one could question above the stability of the nuclear matter against the occurrence of a σ-ω condensate.

We wish to establish in this paper the limits the stability imposes on the model parameters.

2 General formalism

To shortly remember the QHD scheme [1], we introduce the lagrangian density

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_I \tag{1}$$

with

$$\begin{aligned}
\mathcal{L}_N &= \overline{\psi}(i\,\not\partial - m)\psi && (2)\\
\mathcal{L}_\sigma &= \frac{1}{2}(\partial^\mu \sigma)^2 - \frac{1}{2}m_\sigma^2 \sigma^2 && (3)\\
\mathcal{L}_\omega &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2 \omega_\mu \omega^\mu && (4)\\
\mathcal{L}_I &= g_\sigma \overline{\psi}\sigma\psi - g_\omega \overline{\psi}\gamma_\mu \psi \omega^\mu - \frac{1}{4!}a_4 \sigma^4 && (5)
\end{aligned}$$

and

$$F^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \ . \tag{6}$$

We get rid of the non-vanishing σ and ω^0 fields in the vacuum by means of the shifts $\sigma = \sigma' - \bar{\sigma}$, $\omega^\mu \to \omega^{\mu\prime} - \bar{\omega}^\mu$, the mean fields being determined by solving the classical equations of motion:

$$\begin{aligned}
m_\sigma^2 \bar{\sigma} + \frac{a_4}{3!}\bar{\sigma}^3 &= g_\sigma < \overline{\psi}\psi > && (7a)\\
m_\omega^2 \bar{\omega}^0 &= g_\omega < \overline{\psi}\gamma_0 \psi > && (7b)\\
\bar{\omega}^i &= 0 && (7c)
\end{aligned}$$

Their solutions read

$$\bar{\omega}^\mu = \frac{g_\omega \rho}{m_\omega^2}\delta^{\mu 0} \tag{8}$$

and

$$\bar{\sigma} = -\frac{2m_\sigma^2}{\sqrt{a_4}R_\sigma} + \frac{R_\sigma}{\sqrt{a_4}} \ , \tag{9}$$

where ρ is the usual nuclear density

$$\rho = \frac{2k_F^3}{3\pi^2} \tag{10}$$

and

$$R_\sigma = \sqrt[3]{3\sqrt{a_4}g_\sigma \rho_\sigma + \sqrt{8m_\sigma^6 + 9a_4 g_\sigma^2 \rho_\sigma^2}} \tag{11}$$

with

$$\rho_\sigma = -i\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4} S_H(k) \ . \tag{12}$$

In the latter definition S_H denotes a fermion propagator analogous to the nucleon propagator in the medium

$$S^0(k) = \frac{\not k + m}{2E_k}\left\{\frac{\theta(k-k_F)}{k_0 - E_k + i\eta} + \frac{\theta(k_F - k)}{k_0 - E_k - i\eta} - \frac{1}{k_0 + E_k - i\eta}\right\} \tag{13}$$

but with the nucleon mass replaced by an effective one, defined by

$$m^* = m - \bar\sigma \ . \tag{14}$$

Eqs. (9) and (11) need to be solved self-consistently in $\bar\sigma$. Its knowledge immediately provides the effective nucleon mass. It is interesting to note that a self-consistent calculation realizes the prediction of Lee and Wick [2] that for large k_F the nucleon mass is vanishing.

3 The Random Phase Approximation (RPA)

It has recently been observed[3] that the presence of a nuclear medium couples the σ to the 0^{th} component of the ω, so that the σ and ω propagation in the medium will be described by two coupled Dyson equations. We can write them (in momentum space) as a single equation in a 5-dimensional space in the form

$$D^{(5)} = D_0^{(5)} + D_0^{(5)}\Pi^{*(5)}D^{(5)} \tag{15}$$

where $D_0^{(5)}$, $D^{(5)}$ and $\Pi^{*(5)}$ are 5×5 matrices having the structure

$$D_0^{(5)} = \begin{pmatrix} D_{0\sigma} & 0 \\ 0 & D_{0\omega}^{\mu\nu}\end{pmatrix} \quad D^{(5)} = \begin{pmatrix} D_{\sigma\sigma} & D_{\sigma\omega}^{\mu} \\ D_{\omega\sigma}^{\nu} & D_{\omega\omega}^{\mu\nu}\end{pmatrix} \quad \Pi^{*(5)} = \begin{pmatrix} \Pi_{\sigma\sigma}^{*} & \Pi_{\sigma\omega}^{*\mu} \\ \Pi_{\omega\sigma}^{*\nu} & \Pi_{\omega\omega}^{*\mu\nu}\end{pmatrix} \ . \tag{16}$$

The free meson propagators are defined as

$$\begin{aligned} D_{0\sigma}(q) &= g_\sigma \frac{1}{q^2 - m_\sigma^2 + i\epsilon} g_\sigma \ , \qquad (17)\\ D_{0\omega}^{\mu\nu}(q) &= -g_\omega \frac{g^{\mu\nu} - \dfrac{q^\mu q^\nu}{q^2}}{q^2 - m_\omega^2 + i\epsilon} g_\omega \equiv -\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) D_{0\omega}(q) \ . \qquad (18)\end{aligned}$$

The polarisation propagators are constrained by current conservation. Thus they must take the form

$$\Pi^{\mu}_{\sigma\omega}(q) = \Pi^{0}_{\sigma\omega}(q)\mathfrak{N}^{\mu} \equiv \Pi^{V}(q)\mathfrak{N}^{\mu} \ , \tag{19}$$

with

$$\mathfrak{N}^{\mu} = \left(1, \frac{q^0 q^i}{\mathbf{q}^2}\right) \ , \tag{20}$$

and

$$\Pi^{\mu\nu}_{\omega\omega}(q) = \left(\begin{array}{c|c} \Pi^L & \frac{q_0 q_i}{|\mathbf{q}|^2}\Pi^L \\ \hline \frac{q_0 q_j}{|\mathbf{q}|^2}\Pi^L & \frac{q_0^2}{|\mathbf{q}|^2}\Pi^L \frac{q_i q_j}{|\mathbf{q}|^2} + \frac{1}{2}\Pi^T\left(\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}\right) \end{array}\right) \ , \tag{21}$$

with

$$\Pi^L = \Pi^{00}_{\omega\omega}(q) \tag{22}$$

and

$$\Pi^T = \left(\delta^{ij} - \frac{q^i q^j}{|\mathbf{q}|^2}\right)\Pi^{ij}_{\omega\omega}(q) \ . \tag{23}$$

The quantity $\Pi^*_{\sigma\sigma}(q)$ has no tensor structure. We shall define

$$\Pi^S = \Pi^*_{\sigma\sigma} \tag{24}$$

in order to harmonise the notations.

Further, since $q_\mu D^{\mu}_{\sigma\omega} = q_\mu D^{\mu}_{\omega\sigma} = 0$ and $q_\mu D^{\mu\nu}_{\omega\omega} = 0$, as Dyson equation implies, we must also have

$$D^{\mu}_{\sigma\omega} = D^{\mu}_{\omega\sigma} = D^V \mathfrak{N}^{\mu} \tag{25}$$

and

$$D^{\mu\nu}_{\omega\omega} = \left(\begin{array}{c|c} D^L & \frac{q_0 q_i}{|\mathbf{q}|^2}D^L \\ \hline \frac{q_0 q_j}{|\mathbf{q}|^2}D^L & \frac{q_0^2}{|\mathbf{q}|^2}D^L \frac{q_i q_j}{|\mathbf{q}|^2} + \frac{1}{2}D^T\left(\delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}\right) \end{array}\right) \ . \tag{26}$$

The polarisation propagators needs to be approximated in some way. It has been proved in [4] that at the level of mean field in a bosonic space they must be replaced with their zero-order approximation. Thus we replace Π^* with Π^0, with components (traces refer both to spin and isospin)

$$\Pi_{0\sigma\sigma}(q) = -i\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4} S_H(p) S_H(p+q) \ , \tag{27}$$

$$\Pi^{\mu}_{0\sigma\omega}(q) = -i\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4} S_H(p) S_H(p+q)\gamma^{\mu} \tag{28}$$

and

$$\Pi^{\mu\nu}_{0\omega\omega}(q) = -i\mathrm{Tr}\int \frac{d^4p}{(2\pi)^4} S_H(p)\gamma^{\mu} S_H(p+q)\gamma^{\nu} \ . \tag{29}$$

The 0^{th} order propagators $\Pi_0^{S(VLT)}$ have been extensively studied in ref. [5], where their analytical representation is given. Here we observe that they can be written as

$$\begin{aligned}
\Pi_0^S &= 4(4m^2 - q^2)\Pi^0 + 8\mathfrak{T} &(30)\\
\Pi_0^V &= 16m\mathfrak{Q}^V &(31)\\
\Pi_0^L &= 16\mathfrak{Q}^L - 4|\mathbf{q}|^2\Pi^0 + 8\frac{|\mathbf{q}|^2}{q^2}\mathfrak{T} &(32)\\
\Pi_0^T &= 16\mathfrak{Q}^T + 8q^2\Pi^0 - 16\mathfrak{T} \ , &(33)
\end{aligned}$$

where

$$\begin{aligned}
\Pi^0 &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{2E_{\mathbf{p}}}\frac{\theta(k_F-p)}{(\tilde{q}_0+E_{\mathbf{p}})^2 - E^2_{\mathbf{p+q}}}\Bigg|_{p_0=E_{\mathbf{p}}} + (q_0 \longleftrightarrow -q_0) \ , &(34)\\
\mathfrak{Q}^V &= \int \frac{d^3p}{(2\pi)^3}\frac{t^0}{2E_{\mathbf{p}}}\frac{\theta(k_F-p)}{(\tilde{q}_0+E_{\mathbf{p}})^2 - E^2_{\mathbf{p+q}}}\Bigg|_{p_0=E_{\mathbf{p}}} + (q_0 \longleftrightarrow -q_0) \ , &(35)\\
\mathfrak{Q}^L &= \int \frac{d^3p}{(2\pi)^3}\frac{(t^0)^2}{2E_{\mathbf{p}}}\frac{\theta(k_F-p)}{(\tilde{q}_0+E_{\mathbf{p}})^2 - E^2_{\mathbf{p+q}}}\Bigg|_{p_0=E_{\mathbf{p}}} + (q_0 \longleftrightarrow -q_0) \ , &(36)\\
\mathfrak{Q}^T &= \int \frac{d^3p}{(2\pi)^3}\frac{|\mathbf{p}|^2|\mathbf{q}|^2 - (p\cdot\mathbf{q})^2}{2E_{\mathbf{p}}|\mathbf{q}|^2}\frac{\theta(k_F-p)}{(\tilde{q}_0+E_{\mathbf{p}})^2 - E^2_{\mathbf{p+q}}}\Bigg|_{p_0=E_{\mathbf{p}}} + (q_0 \longleftrightarrow -q_0) \ , &(37)\\
\mathfrak{T} &= \int \frac{d^3p}{(2\pi)^3}\frac{\theta(k_F-p)}{2E_{\mathbf{p}}} \ . &(38)
\end{aligned}$$

In the above we have introduced the transverse vector

$$t^{\mu} = p^{\mu} - \frac{p\cdot q}{q^2}q^{\mu} \ , \tag{39}$$

while

$$\tilde{q}_0 = q_0 + i\eta \,\mathrm{sign}(q_0) \tag{40}$$

accounts for the right analytical determination near the cuts.

We can now solve the Dyson equation (15), finding

$$D_{\sigma\sigma} = \widetilde{D}_\sigma \frac{1}{1 - \Pi_0^V \widetilde{D}_\omega \Pi_0^V \widetilde{D}_\sigma} \tag{41}$$

$$D^L = \left(\frac{|\mathbf{q}|^2}{q^2}\right)^2 \widetilde{D}_\omega \frac{1}{1 - \Pi_0^V \widetilde{D}_\sigma \Pi_0^V \widetilde{D}_\omega} \tag{42}$$

$$D^V = -\frac{|\mathbf{q}|^2}{q^2} D_{\sigma\sigma} \Pi_0^V \widetilde{D}_\omega \;=\; -\frac{q^2}{|\mathbf{q}|^2} \widetilde{D}_\sigma \Pi_0^V D^L \tag{43}$$

$$D^T = \frac{2D_{0\omega}}{1 - \frac{1}{2} D_{0\omega} \Pi_0^T} \;, \tag{44}$$

where

$$\widetilde{D}_\sigma = \frac{D_{0\sigma}}{1 - D_{0\sigma} \Pi_0^S} \tag{45}$$

$$\widetilde{D}_\omega = \frac{\frac{q^2}{|\mathbf{q}|^2} D_{0\omega}}{1 - \frac{q^2}{|\mathbf{q}|^2} D_{0\omega} \Pi_0^L} \;. \tag{46}$$

Observe that the transverse part of the ω-meson is completely decoupled from the longitudinal propagation, that instead involves σ and ω^0.

3.1 The σ-ω condensate

We consider first a simplified model where only the σ meson exists. Since the σ-exchange is attractive, a σ condensation is expected under certain conditions. In this model the RPA σ propagator (41) reads

$$D_{\sigma\sigma} = \frac{D_{0\sigma}}{1 - D_{0\sigma} \Pi_0^S} \tag{47}$$

and a σ condensate arises if the denominator of the above equation vanishes at a certain $|\mathbf{q}|$ and at $q_0 = 0$, i.e., if the equation

$$1 - D_{0\sigma}(|\mathbf{q}|, q_0 = 0) \Pi_0^S(|\mathbf{q}|, q_0 = 0) = 0 \tag{48}$$

admits some solution in $|\mathbf{q}|$.

Let us now study the function $D_{0\sigma}(|\mathbf{q}|, q_0 = 0)\Pi_0^S(|\mathbf{q}|, q_0 = 0)$. From eq. (17) we get

$$D_{0\sigma}(|\mathbf{q}|, q_0 = 0) = -\frac{g_\sigma^2}{|\mathbf{q}|^2 + m_\sigma^2} ,$$

while an easy calculation, using the explicit expressions given in [5], provides

$$\Pi_0^S(|\mathbf{q}|, q_0 = 0) = -\frac{4m^2 + |\mathbf{q}|^2}{2\pi^2|\mathbf{q}|}\Bigg\{|\mathbf{q}|\log\frac{k_F + E_F}{m} + E_F\log\left|\frac{2k_F + |\mathbf{q}|}{2k_F - |\mathbf{q}|}\right| - \frac{1}{2}\sqrt{4m^2 + |\mathbf{q}|^2}\log\left|\frac{E_F|\mathbf{q}| + k_F\sqrt{4m^2 + |\mathbf{q}|^2}}{E_F|\mathbf{q}| - k_F\sqrt{4m^2 + |\mathbf{q}|^2}}\right|\Bigg\} . \tag{49}$$

A simple check shows that this function is regular at $|\mathbf{q}| = 2k_F$, and from the above it immediately follows that

$$\lim_{|\mathbf{q}|\to 0} D_{0\sigma}(|\mathbf{q}|, q_0 = 0)\Pi_0^S(|\mathbf{q}|, q_0 = 0) = \frac{g_\sigma^2}{m_\sigma^2}\frac{2m^2}{\pi^2}\log\frac{k_F + E_F}{m} > 0 . \tag{50}$$

On the other hand for large $|\mathbf{q}|$'s $\Pi_0^S(|\mathbf{q}|, q_0 = 0)$ has a finite limit so that

$$D_{0\sigma}(|\mathbf{q}|, q_0 = 0)\Pi_0^S(|\mathbf{q}|, q_0 = 0) \underset{|\mathbf{q}|\to\infty}{\longrightarrow} |\mathbf{q}|^{-2} . \tag{51}$$

As a consequence eq. (48) has certainly a solution provided

$$\frac{g_\sigma^2}{m_\sigma^2}\frac{2m^2}{\pi^2}\log\frac{k_F + E_F}{m} > 1 . \tag{52}$$

The physics contained in this conclusion was expected: a sufficiently large attraction produces a σ condensate. Numerically at the normal nuclear density ($k_F = 1.36\text{fm}^{-1}$) and with $m_\sigma = 550$ MeV used in QHD [1] we get for g_σ the critical value $g_\sigma = 2.47$, well below the value of the QHD, namely $g_\sigma = 9.573$.

Actually the ω-meson exchange, being repulsive, could prevent such an occurrence. A simple algebra shows that $D_{\sigma\sigma}$, D^L and D^V, as given in eqs. (41), (42) and (43), have the same denominator, namely

$$\mathfrak{D}_L = \left[1 - D_{0\sigma}\Pi^S\right]\left[1 - (\zeta - 2)D_{0\omega}\Pi^L\right] - (\zeta - 2)D_{0\sigma}D_{0\omega}\left(\Pi^V\right)^2 \tag{53}$$

(the index L reminds us that we are concerned with the longitudinal propagation), where we have introduced the shortcut

$$\zeta = \frac{|\mathbf{q}|^2 + q_0^2}{|\mathbf{q}|^2} \qquad \Longrightarrow \qquad \zeta - 2 = \frac{q^2}{|\mathbf{q}|^2} \ . \tag{54}$$

The occurrence of a phase transition is now signalled by the existence of solutions of the equation

$$\mathfrak{D}_L = 0 \ . \tag{55}$$

Proceeding as before we find

$$\Pi^V(|\mathbf{q}|, q_0 = 0) = -\frac{m}{(2\pi)^2|\mathbf{q}|}\left\{4k_F|\mathbf{q}| + (4k_F^2 - |\mathbf{q}|^2)\log\left|\frac{2k_F + |\mathbf{q}|}{2k_F - |\mathbf{q}|}\right|\right\} \tag{56}$$

$$\begin{aligned}\Pi^L(|\mathbf{q}|, q_0 = 0) = -\frac{1}{6\pi^2|\mathbf{q}|}\Bigg\{ & 2|\mathbf{q}|k_F E_F + 2|\mathbf{q}|(3m^2 - |\mathbf{q}|^2)\log\frac{k_F + E_F}{m} \\ & + E_F(4E_F^2 - 3|\mathbf{q}|^2)\log\left|\frac{2k_F + |\mathbf{q}|}{2k_F - |\mathbf{q}|}\right| \\ & - \sqrt{4m^2 + |\mathbf{q}|^2}(2m^2 - |\mathbf{q}|^2)\log\left|\frac{E_F|\mathbf{q}| + k_F\sqrt{4m^2 + |\mathbf{q}|^2}}{E_F|\mathbf{q}| - k_F\sqrt{4m^2 + |\mathbf{q}|^2}}\right|\Bigg\}\end{aligned} \tag{57}$$

together with the relevant limits for $|\mathbf{q}| \to 0$

$$\lim_{|\mathbf{q}|\to 0} \Pi^V(|\mathbf{q}|, q_0 = 0) = -\frac{2mk_F}{\pi^2} \tag{58}$$

$$\lim_{|\mathbf{q}|\to 0} \Pi^L(|\mathbf{q}|, q_0 = 0) = -\frac{1}{\pi^2}\left\{k_F E_F + m^2\log\frac{k_F + E_F}{m}\right\} \ , \tag{59}$$

both limits being negative, and for $|\mathbf{q}| \to \infty$

$$\Pi^V(|\mathbf{q}|, q_0 = 0) \sim \frac{1}{|\mathbf{q}|^2} \tag{60}$$

$$\Pi^L(|\mathbf{q}|, q_0 = 0) \to \text{constant} \ . \tag{61}$$

Moreover in the limit $q_0 \to 0$ we find

$$\zeta = 1 \ , \qquad\qquad \zeta - 2 = -1 \ .$$

Thus at $q_0 = 0$

$$\mathfrak{D}_L \Big|_{q_0=0} = \left[1 - D_{0\sigma}\Pi^S\right]\left[1 + D_{0\omega}\Pi^L\right] + D_{0\sigma}D_{0\omega}\left(\Pi^V\right)^2 \ . \tag{62}$$

which implies, using (18), (51), (60) and (61)

$$\lim_{|\mathbf{q}|\to\infty} \mathfrak{D}_L(|\mathbf{q}|, q_0 = 0) = 1 \ . \tag{63}$$

Thus a condensed state will occur if

$$\lim_{|\mathbf{q}|\to 0} \mathfrak{D}_L(|\mathbf{q}|, q_0 = 0) \leq 0 \ . \tag{64}$$

Let now consider the above limit as a function of k_F:

$$\phi(k_F) = \lim_{|\mathbf{q}|\to 0} \mathfrak{D}_L(|\mathbf{q}|, q_0 = 0) \ . \tag{65}$$

From (51),(58) and (59) we derive, for low k_F,

$$\phi(k_F) = 1 - \frac{2m}{\pi^2}\left(\frac{g_\sigma^2}{m_\sigma^2} - \frac{g_\omega^2}{m_\omega^2}\right)k_F + \mathcal{O}(k_F^2) \ , \tag{66}$$

while at large k_F we find the asymptotic behaviour

$$\phi(k_F) \simeq -\frac{2}{\pi^4}\frac{g_\sigma^2}{m_\sigma^2}\frac{g_\omega^2}{m_\omega^2}m^2k_F^2\log\frac{2k_F}{m} + \frac{1}{\pi^4}\frac{g_\omega^2(4g_\sigma^2m^2 + \pi^2m_\sigma^2)}{m_\sigma^2m_\omega^2}k_F^2 \ . \tag{67}$$

Hence $\phi(k_F)$ goes to 1 for low k_F and to $-\infty$ for large k_F and thus a critical value for k_F surely exists. The problem of how and when this critical point is reached and this can be addressed only numerically.

Before exploiting the calculations, we observe that up to now we have not accounted for self-consistency in determining the nucleon mass, according to eq. (14). The nucleon effective mass m^* is displayed in fig. 1 and compared with a non-self-consistent calculation (i.e., with S_H replaced by S_0 in eq. (12)). The plot shows, as pointed out by Lee and Wick many years ago[2], that the self-consistent effective mass tends to vanish at large densities. Thus the large k_F limit (67) must be evaluated at $m = 0$, namely

$$\phi(k_F) \simeq \frac{g_\omega^2}{m_\omega^2}\frac{k_F^2}{\pi^2} > 0 \ . \tag{68}$$

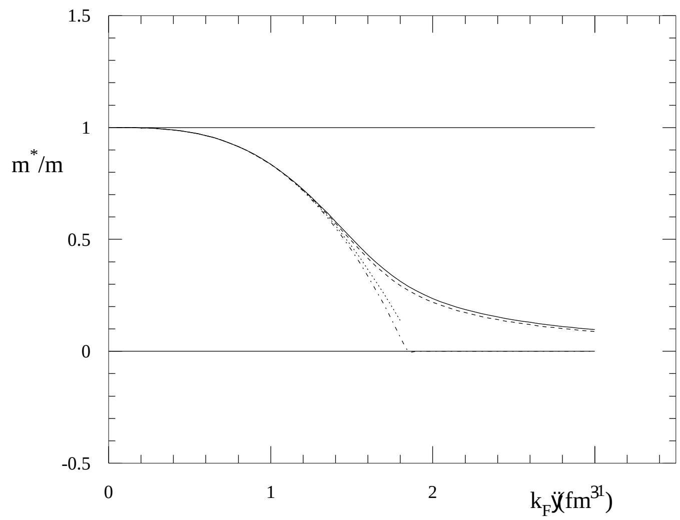

Figure 1: Nucleon effective mass. Solid line: full self-consistent calculation; dashed line: self-consistent calculation with $a_4 = 0$; dotted line: non- self-consistent calculation, dash-dotted line: non- self-consistent calculation with $a_4 = 0$.

On the other hand, at low k_F the effective mass tends to the bare one and with the standard parameters of QHD, namely $g_\sigma = 9.573$, $m_\sigma = 550$ MeV, $g_\omega = 11.67$ and $m_\omega = 783$ MeV, $\phi(k_F)$ start decreasing if

$$\frac{g_\sigma^2}{m_\sigma^2} - \frac{g_\omega^2}{m_\omega^2} > 0 \ .$$

Thus one may reasonably argue that the function may have a minimum below 0. An explicit calculation confirms this guess, as shown in fig. 2.

Actually, to be reasonably sure that no condensation arises, we could require

$$\phi'(k_F) \Big|_{k_F=0} \geq 0$$

that corresponds to a limiting value $g_\sigma = 8.197$, to be compared with $g_\sigma = 9.573$ as given in [1].

A more detailed search for limiting conditions requires a numerical calculation. In fig. 3 we show the behaviour of ϕ for five different values of g_σ

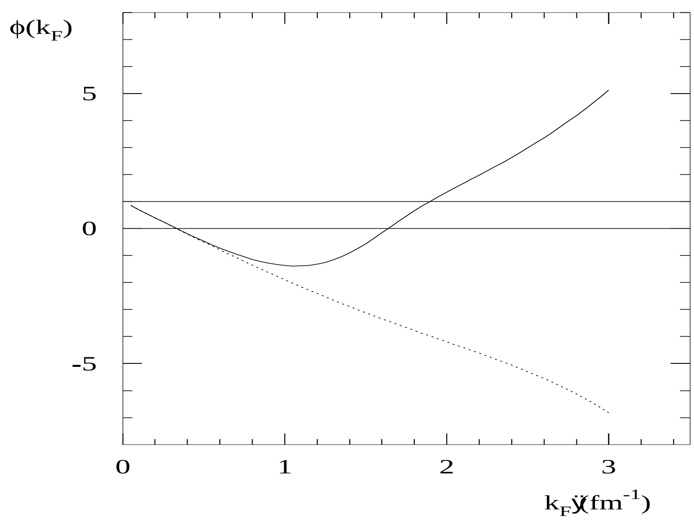

Figure 2: Plot of the function $\phi(k_F)$ with a self-consistent effective mass (solid line) and the bare mass (dashed line). The last is evaluated with the standard values of QHD.

ranging from $g_\sigma = 10$ (lowest solid line) to $g_\sigma = 8$ (upper solid line). It turns out that the critical value for the existence of a σ- condensate is

$$g_{\sigma\,\mathrm{crit}} = 8.828 \ . \tag{69}$$

It is clear nevertheless that close to the critical value precursor phenomena (like in pion condensation) are to be expected, thus a lower value for g_σ should be preferred.

The above states that at $g_\sigma = 8.828$ there is (as the curves in fig. 2 suggest) only one value of k_F at which the σ-condensation occurs. Numerically this happens at $k_F^{\mathrm{crit}} = 207.8$ MeV/c. Above this value we see that two solutions of the equation $\phi(k_F) = 0$ exist. Thus below the lowest solution we find no σ-condensation, and the same occurs above the higher one. In fig. 4 we have plotted the function $\mathfrak{D}_L|_{q_0=0}$ for three different values of k_F. This shows how below the lower value and above the higher one no phase transition seems to occur. Actually this statement is only formal: indeed when a phase transition is found at the lower critical value of k_F, then what hap-

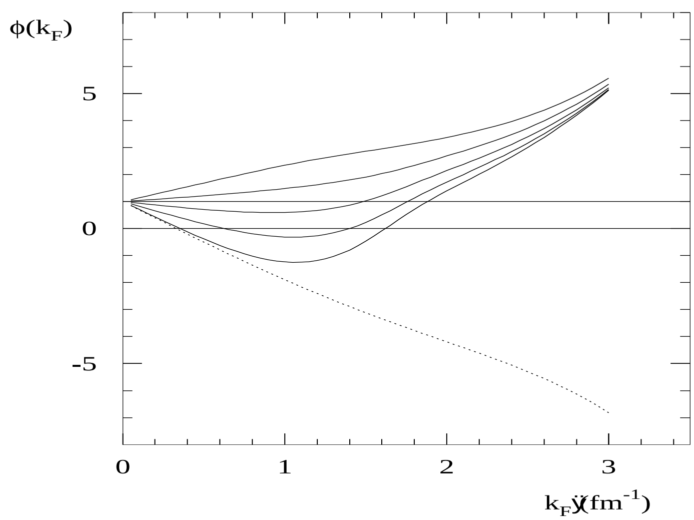

Figure 3: The function $\phi(k_F)$ for different values of g_σ, ranging from 10 (lowest solid line) to 8 (upper one) with a spacing of 0.5, with self-consistent effective mass. The dashed line corresponds to a bare nucleon mass.

pens for higher k_F becomes unpredicable in the present formalism. On the other hand since no evidence exists about this phase transition, the existence of another critical value of k_F is simply of mathematical, but not physical, interest.

4 Conclusions

The QHD was taylored to describe the static properties of the nuclear matter at the mean field level, i.e., at the 0^{th} order in the loop expansion. Within this approximation no phase transition can arise.

We can go beyond, however, and consider an expansion in bosonic loops only (boson loop expansion, in short BLE), as described in [4, 6]. There it was proved that the RPA series is just the mean field of the BLE: it contains in fact fermionic loops but not bosonic ones and in principle it may originate phase transitions, as is well known from the old discussions about

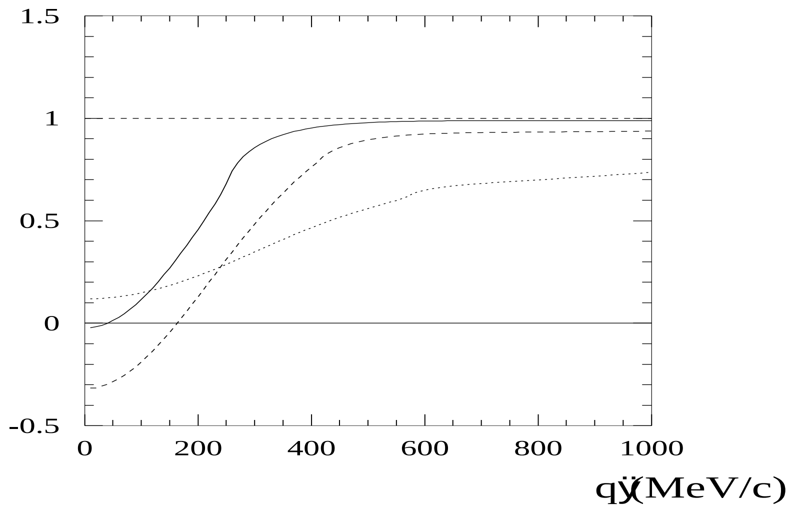

Figure 4: The function $\mathfrak{D}_L|_{q_0=0}$ for different values of k_F. Solid line: $k_F = 127.8$ MeV/c, dashed line: $k_F = 207.8$ MeV/c, dotted line $k_F = 287.8$ MeV/c.

pion condensation.

In this paper we have shown in fact that at BLE-mean field level the Serot and Walecka model would predict the occurrence of a σ-ω condensed phase. This outcome however does not destroy the whole apparatus of QHD: it simply states that the mean field level is not able to provide a stable ground state at least above some critical values of k_F. It is remarkable, nevertheless, that these critical values lie below the normal nuclear density.

To overcome this instability we need to perform a higher order calculation of the binding energy (at the order of two bosonic loops). Since the value of the parameters in this model have been fixed at the mean field level, in going beyond we need a reparametrization, that hopefully should lead to a non-critical choice.

To conclude we observe that in the present model we have studied the possible occurrence of a condensed state as g_σ varies. We could as well modify the σ mass, as the relevant parameter is their ratio. Actually this is ininfluent as far as we look at the existence of a critical point, however, it matters when

we consider the momentum $\mathbf{q}$ where the condensation occurs.

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